Bayesian Analysis and MCMC

Scott A. Baldwin

My Goals

1. Introduce Bayesian ideas and why you might care

2. Demonstration

Goals of Statistical Analysis

1. Estimate Parameters $\mu, \sigma, r, \beta, \lambda$

2. Predictions What will new data look like?

Approaches

Frequentist

Maximum Likelihood

Bayesian

MCMC

Maximum Likelihood

What parameters most likely produced the data?

Flip a coin 10 time:
6 Heads; 4 Tails

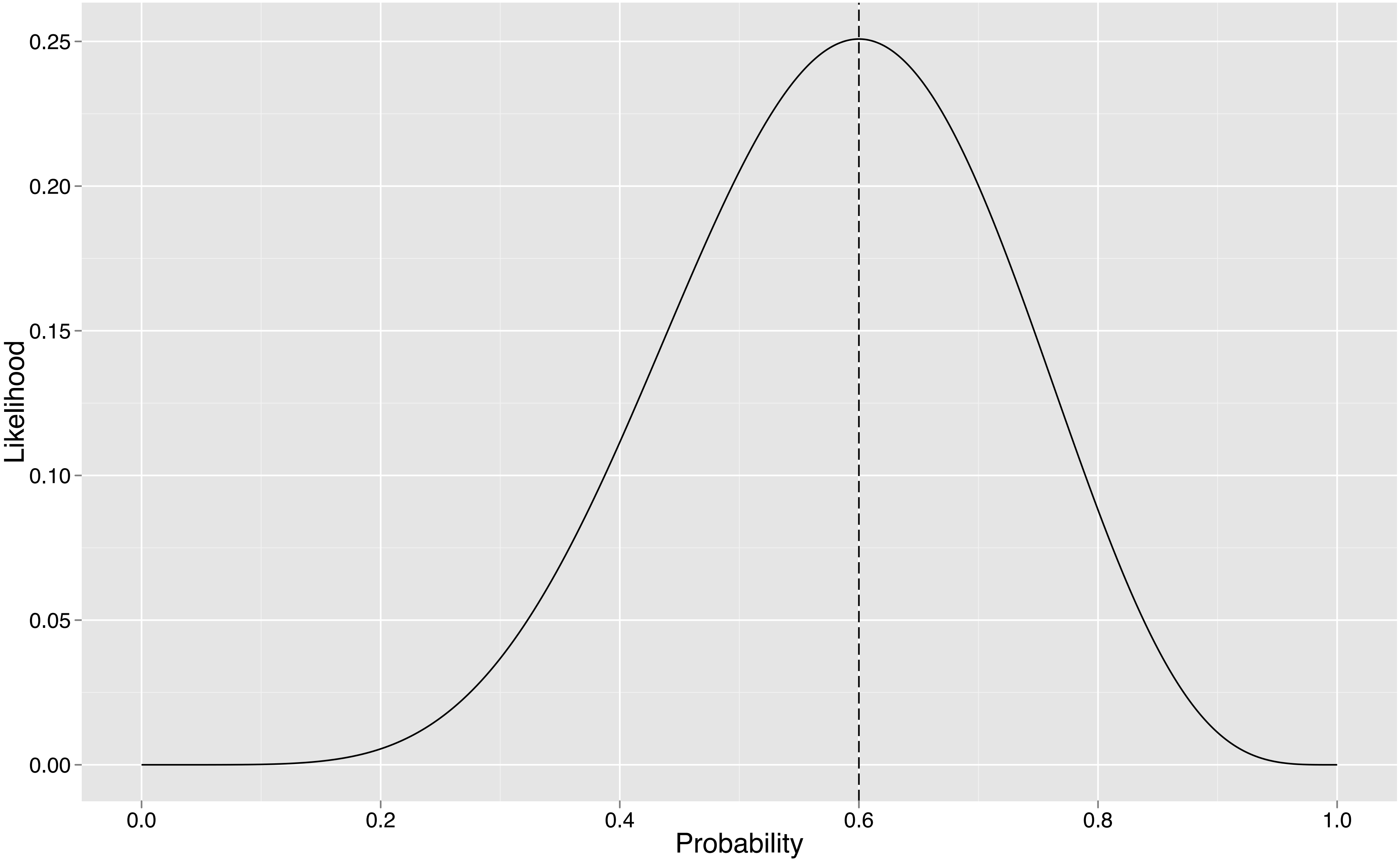
What's the probability
of heads?

$$p(6)=\binom{10}{6}\underset{\nwarrow}{p}^{6}(1-\underset{\nearrow}{p})^{10-6}$$

unknown

$$p(6)=\binom{10}{6}p^{6}(1-p)^{10-6}$$

p = 0.6 is the ML estimate



Goals of Statistical Analysis

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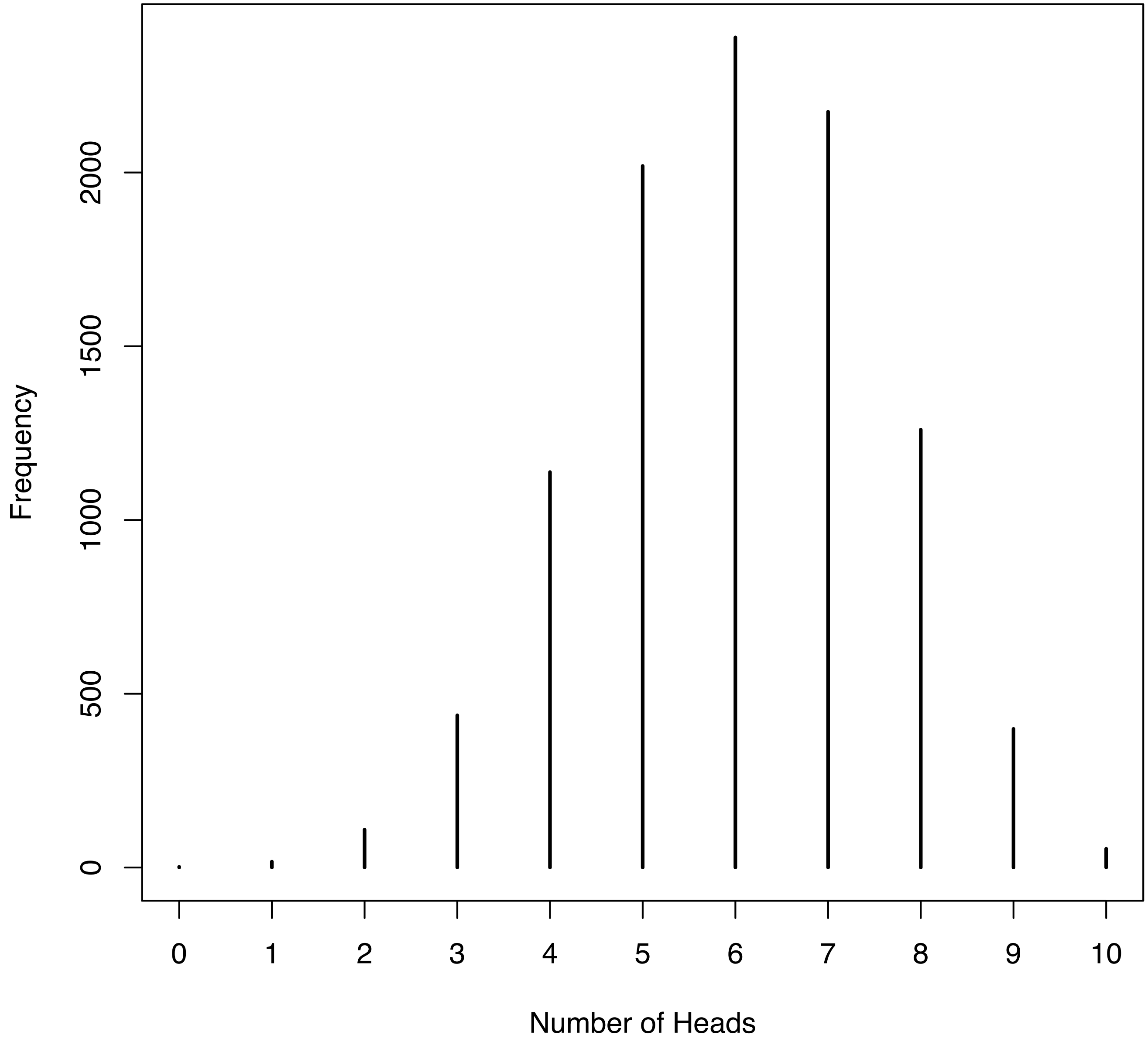
Predicted Data

`rbinom(10000, 10, .6)`



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Bayesian Analysis

Estimate unknowns via
Bayes theorem

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$$p(\text{theory} \mid \text{data}) = \frac{p(\text{data} \mid \text{theory})\, p(\text{theory})}{p(\text{data})}$$

Estimate unknowns via
Bayes theorem

$$\overset{\substack{\text{Posterior}\\ \big\downarrow}}{p\left(\text{theory} \mid \text{data}\right)} = \frac{\overset{\substack{\text{Likelihood}\\ \big\downarrow}}{p\left(\text{data} \mid \text{theory}\right)}\, \overset{\substack{\text{Prior}\\ \big\downarrow}}{p\left(\text{theory}\right)}}{p\left(\text{data}\right)}$$

Priors

Represent Prior Beliefs

Uncertainty about the unknowns prior to seeing the data

Prior for an intraclass correlation

$$\rho = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$$

Prior research suggests most ICCs for group therapy studies range from 0-0.30

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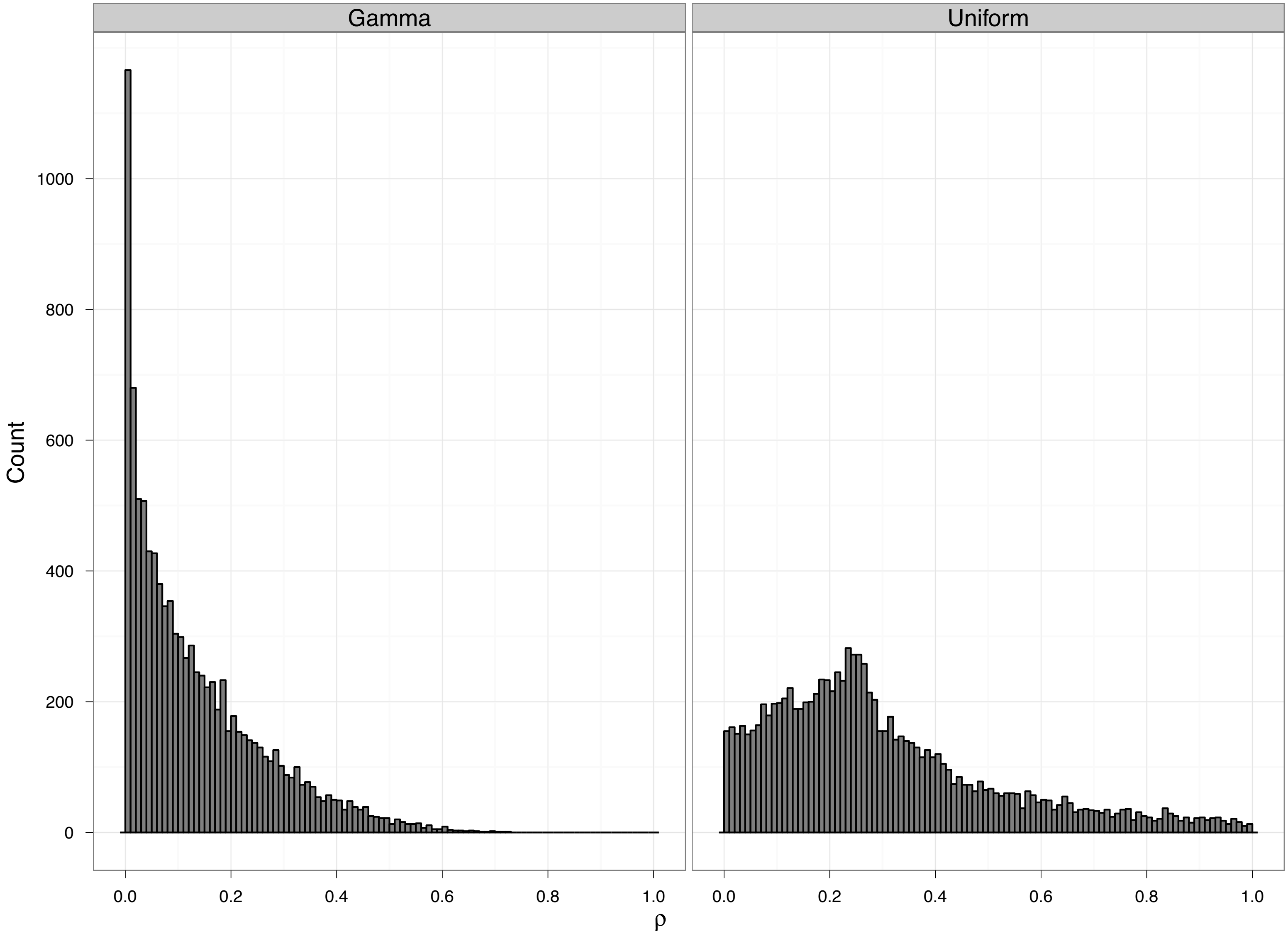
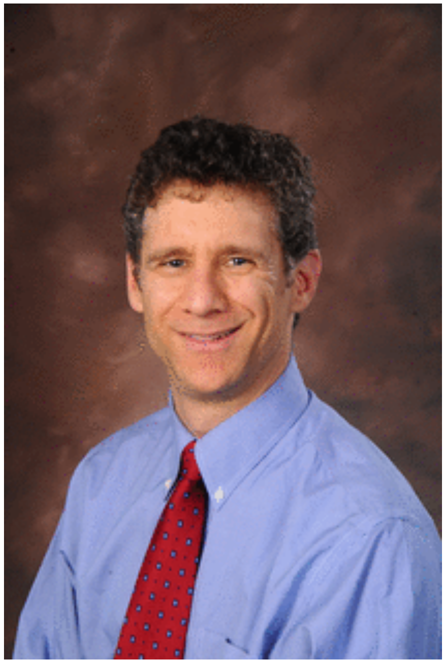
$$\rho = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$$



But I want the data, and only the data, to speak!



“More generally, though, I think we should avoid the temptation to think that, when a Bayesian inference goes wrong, it has to be a problem with the prior. That’s old-fashioned thinking, the idea that the likelihood is God-given and known perfectly, leaving us all to fight over our priors. In many cases, the model matters (for example, in our discussion above about natural-seeming but flawed discrete models). Even if the data model generally makes sense, its details can matter: as I point out to my students, the prior only counts once in the posterior, but the likelihood comes in over and over again, once for each data point.”

http://andrewgelman.com/2012/10/i-notice-a-slightly-garbled-version-of-bayesian-inference-which-provokes-some-thoughts-on-the-applicability-of-bayesian-models-of-human-reasoning/

Estimate unknowns via
Bayes theorem

$$\overset{\text{Posterior}}{\overset{\downarrow}{p\left(\text{theory} \mid \text{data}\right)}} = \frac{\overset{\text{Likelihood}}{\overset{\downarrow}{p\left(\text{data} \mid \text{theory}\right)}}\,\overset{\text{Prior}}{\overset{\downarrow}{p\left(\text{theory}\right)}}}{p\left(\text{data}\right)}$$

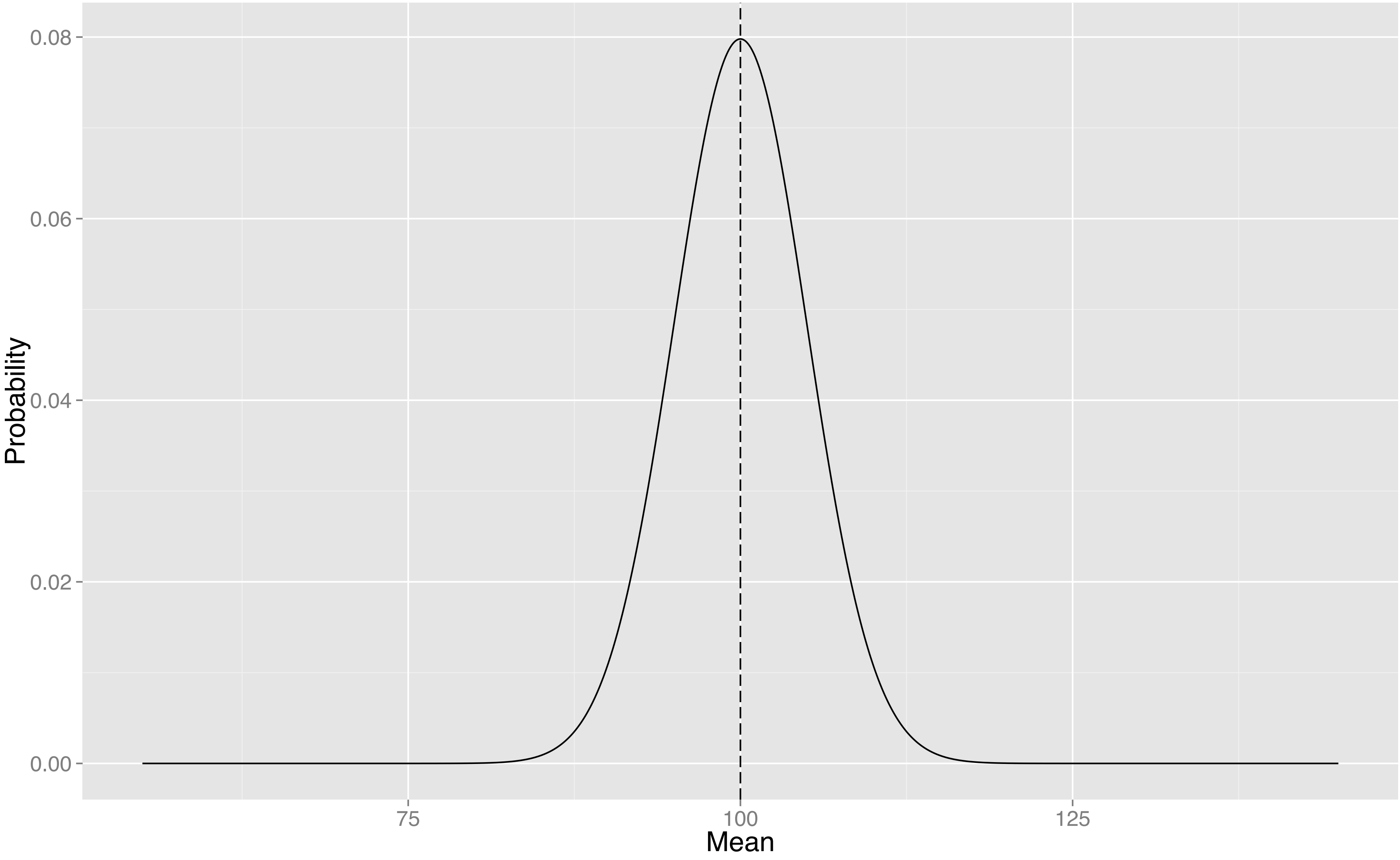
Posterior Distributions

What is a posterior distribution?

Combination of information from:
the data and the prior

A probability distribution for a parameter

Distribution not a point estimate



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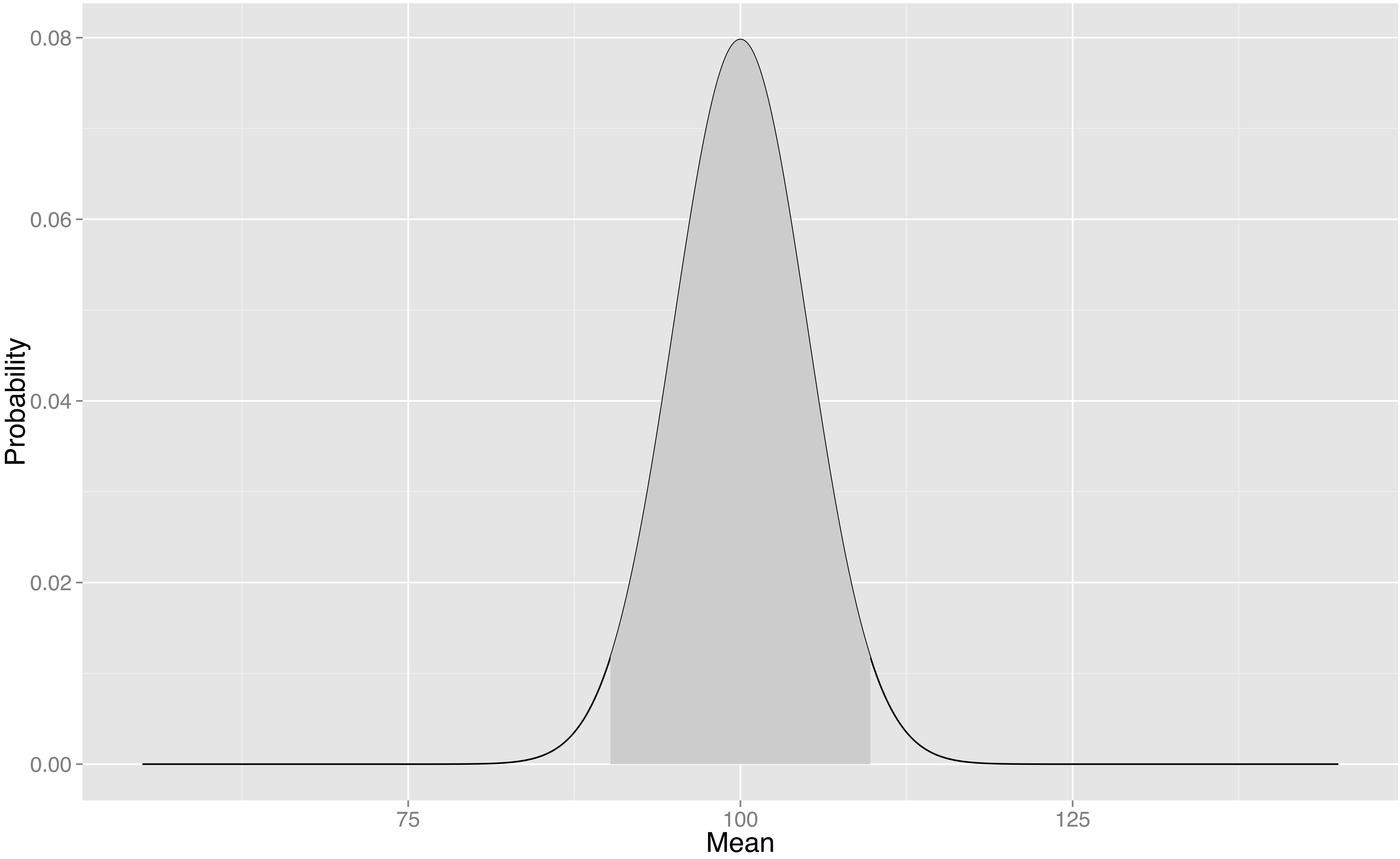


Posterior simulations

If the posterior distribution conforms to a known probability distribution:

- we just use what we know about the probability distribution

If we don't, we use simulation:

Simon Jackman (paraphrasing): Anything we want to know about an unknown parameter can be found by simulating from the distribution of that parameter.

```
tempdata <- rnorm(n=20000,mean=?,sd=?)
```

What’s the mean?   What’s the sd?

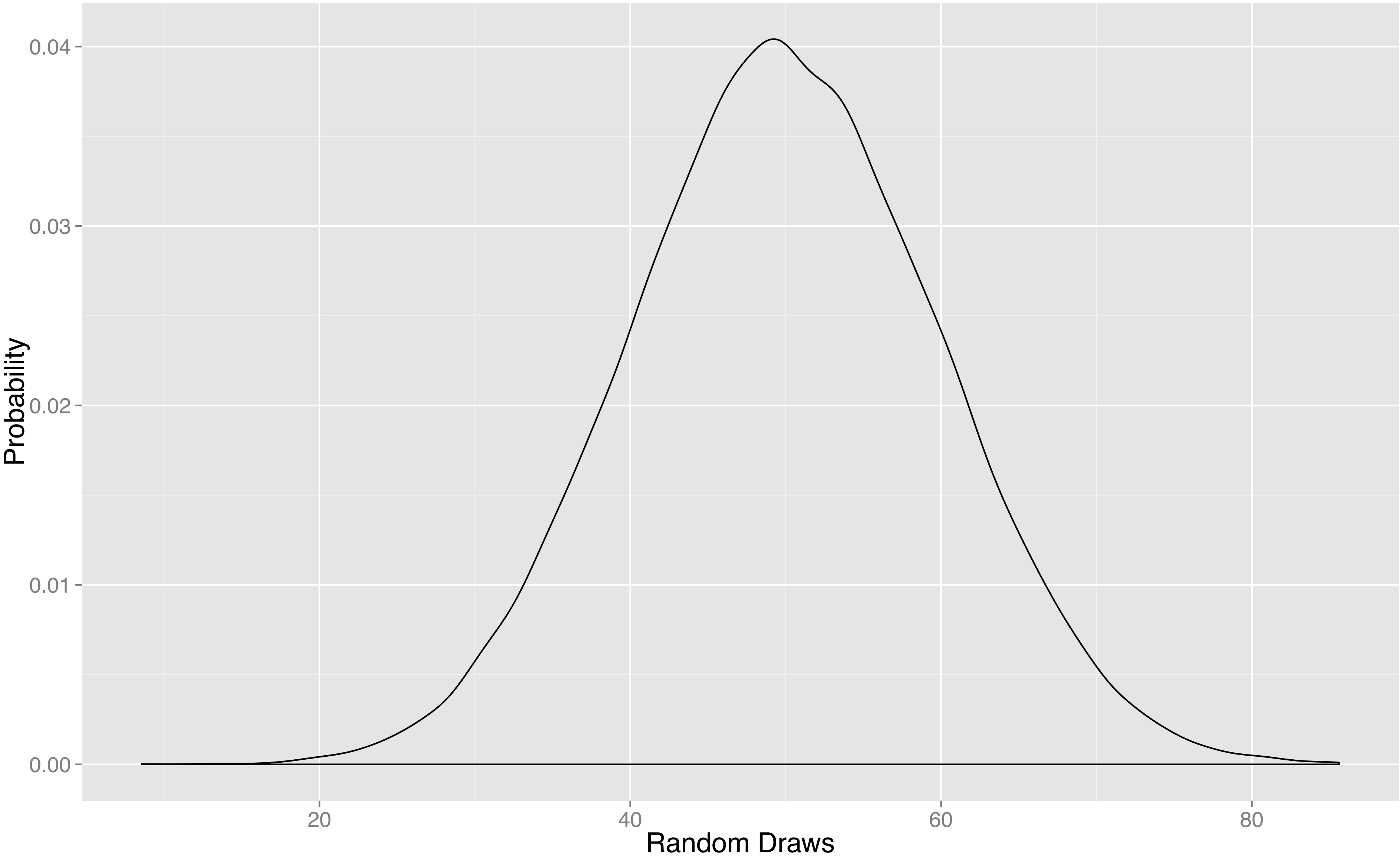
Use the 20,000 draws to learn about the distribution

```
tempdata <- rnorm(n=20000,mean=?,sd=?)
```

```
> mean(tempdata)    > sd(tempdata)
[1] 50.1143         [1] 10.00049
```

```
> quantile(tempdata, c(.025,.975))
    2.5%    97.5%
30.45645 69.79223
```

`tempdata <- rnorm(n=20000,mean=?,sd=?)`



Posterior of Unknown Form

But we have functions for normal distributions, so simulating is easy....What do we do if we don’t know the actual form of the posterior distribution?

Markov Chain Monte Carlo
MCMC

All you (or the computer program) need to know
is the form of the posterior up to a constant

MCMC will produce random draws from the posterior

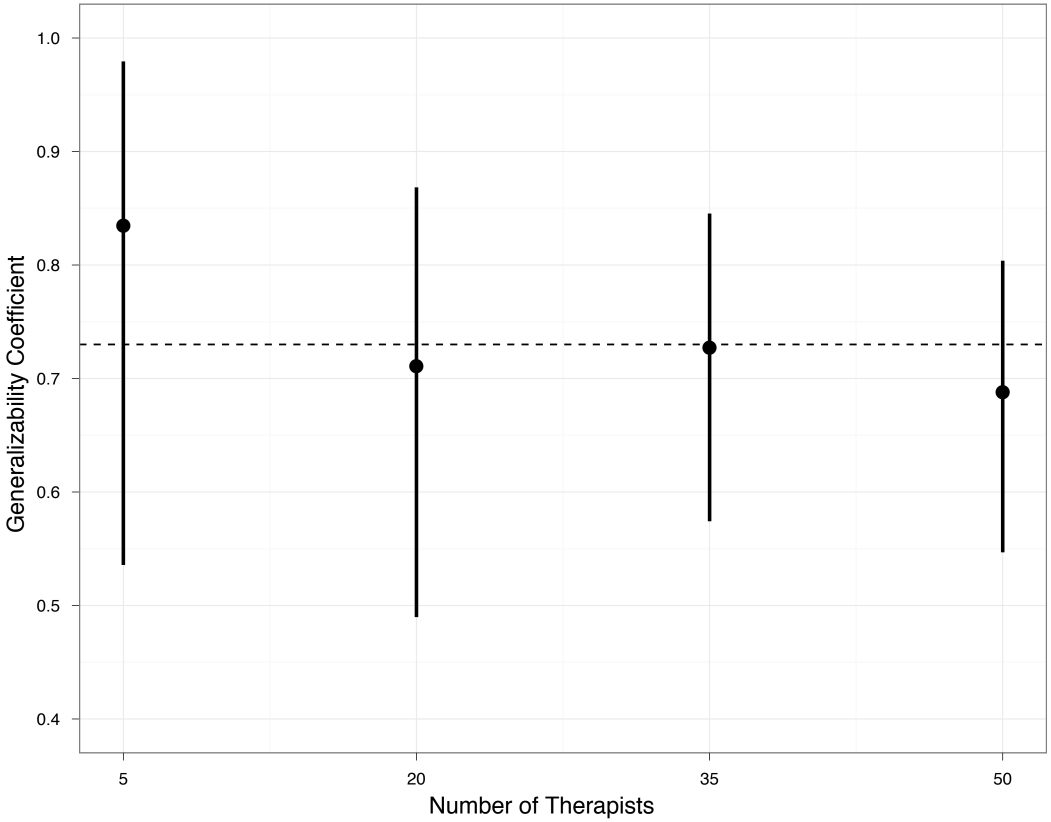
MCMC will produce random draws from the posterior

Can easily obtain posterior distributions for combination of parameters

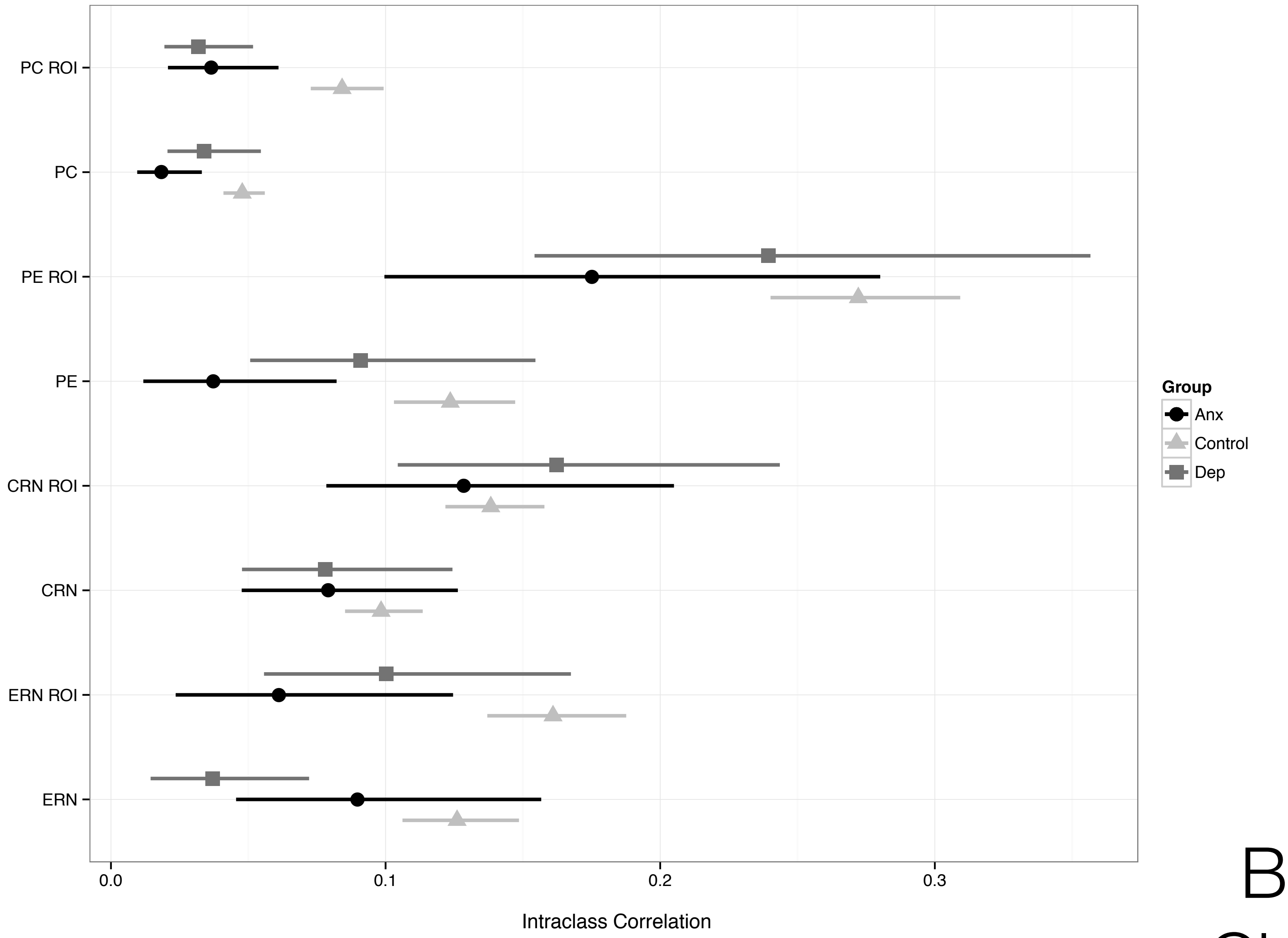
$$\rho = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$$

Can easily obtain posterior distributions for combination of parameters

$$GC_t = \frac{\sigma_t^2}{\sigma_t^2 + \sigma_{p:t,e}^2 / m}$$

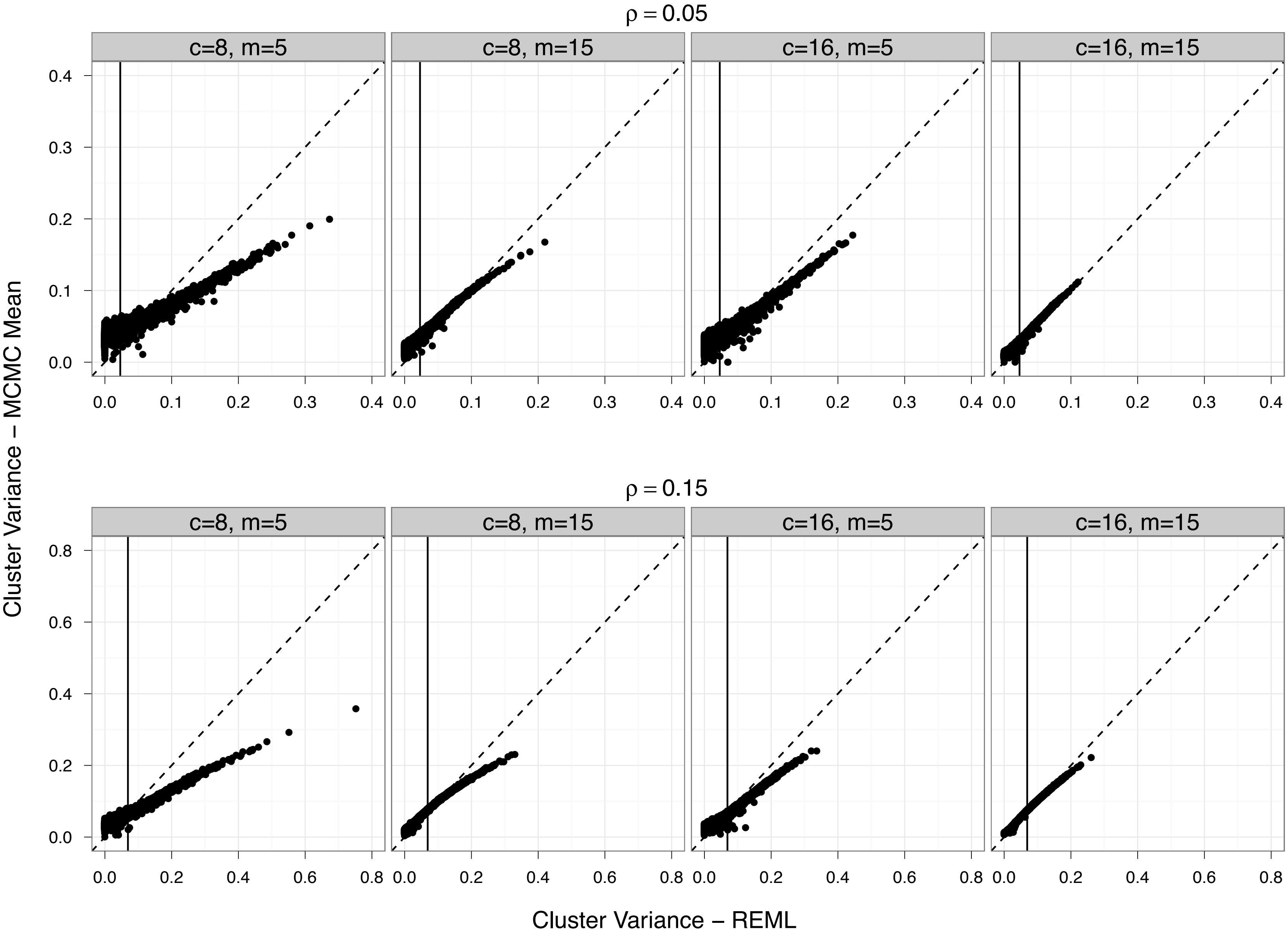


Baldwin, Imel, and Atkins, 2012



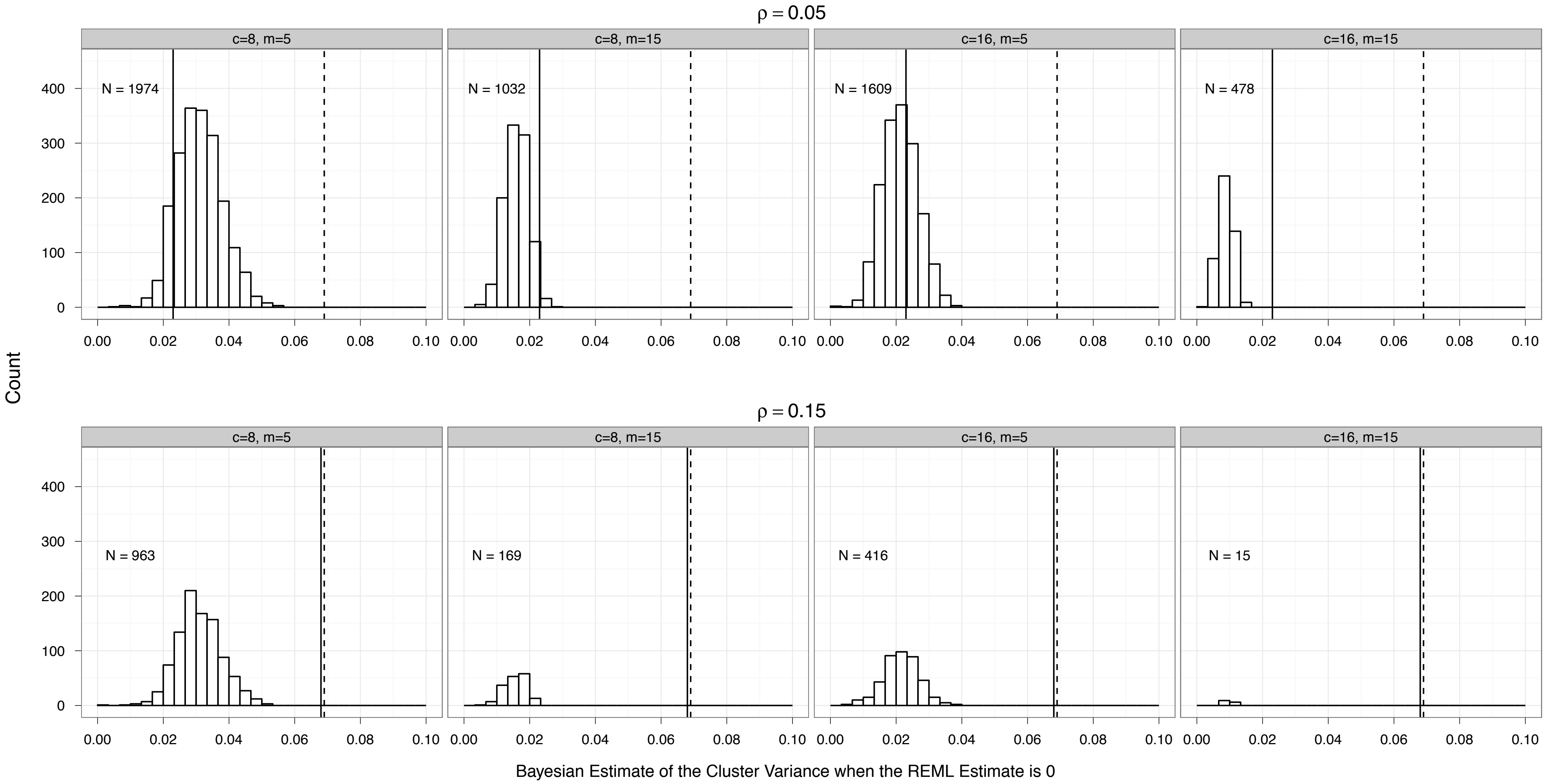
Baldwin, Clayson, & Larson, under review

Priors help stabilize estimates



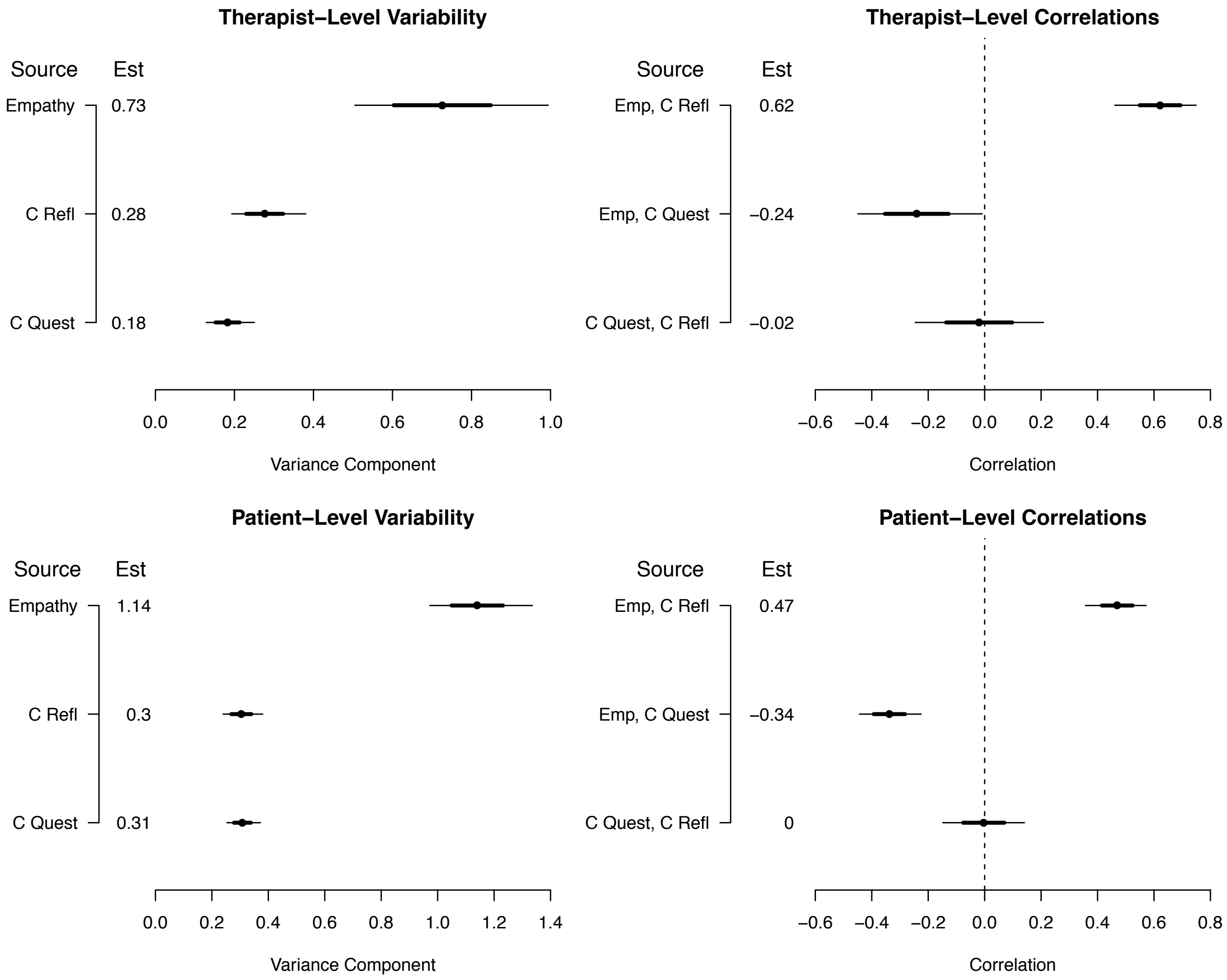
Baldwin & Fellingham, 2013

Priors help give more realistic estimates



Baldwin & Fellingham, 2013

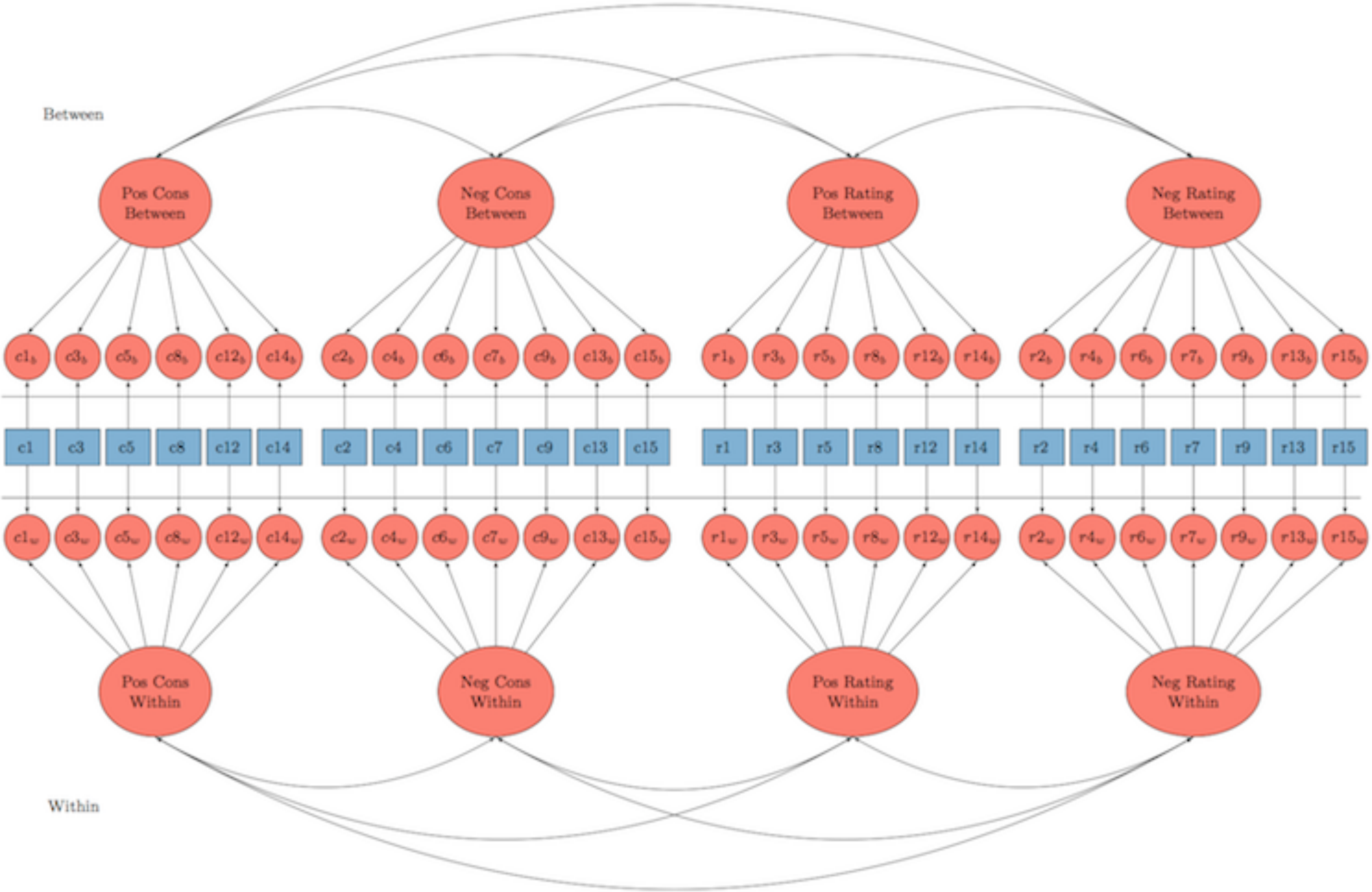
MCMC can be used when ML has difficulty



Therapist Fidelity in MI - MITI

Baldwin, Imel, Braithwaite, & Atkins, in progress

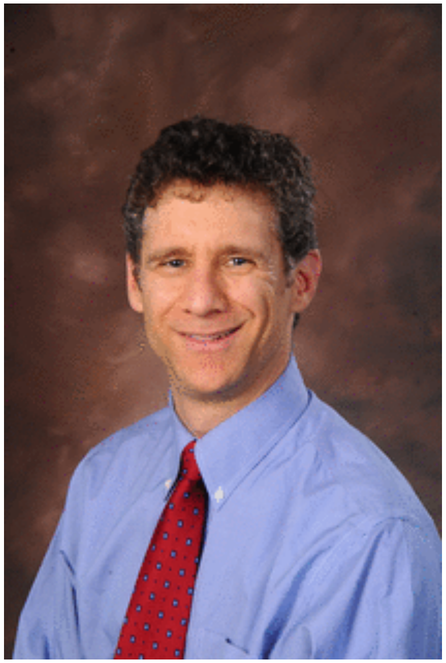
MCMC can be used when ML has difficulty



Swiss
Cheese
Problem

Lee, Baldwin, & Atkins, in progress

But I want the data, and only the data, to speak!



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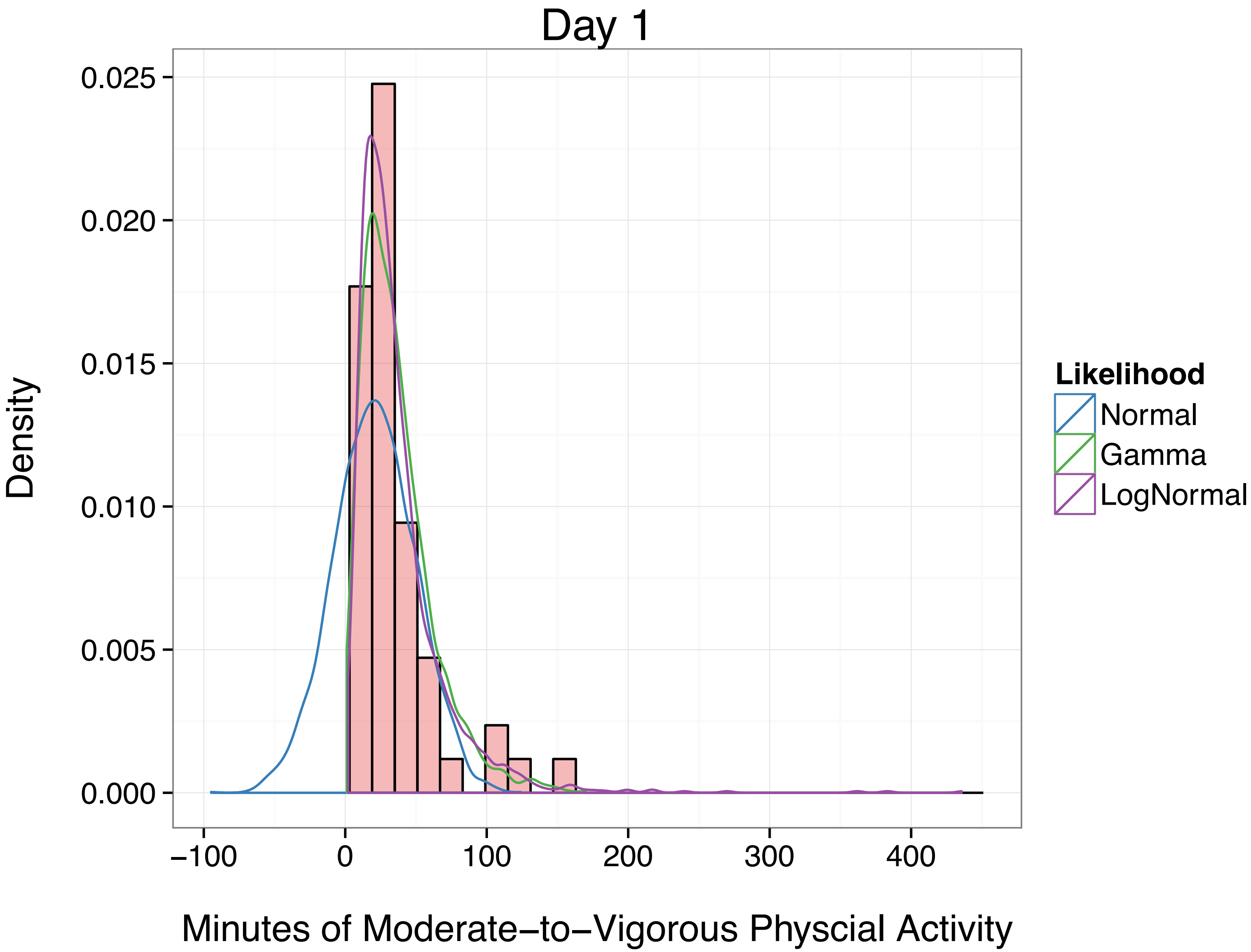
Prediction - Likelihood Matters



Baldwin, Baldwin, & Fellingham, in progress

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Demo

Software

Options

- General purpose MCMC software
  - WinBUGS
  - JAGS
  - STAN
  - PyMC
  - Proc MCMC

```
model {
  for (i in 1:mpath1) {
    fcz1_path[i] ~ dnorm(upath[pathid[i]],ppath)
  }
  for (j in 1:gpath1) {
    upath[j] ~ dnorm(bpath,pupath)
  }

  bpath ~ dnorm(0, .0001)
  varpath ~ dunif(0,100)
  pupath <- 1/varpath
  errpath ~ dunif(0,200)
  ppath <- 1/errpath

  for (i in 1:mcont1) {
    fcz1_cont[i] ~ dnorm(ucont[contid[i]],pcont)
  }
  for (j in 1:gcont1) {
    ucont[j] ~ dnorm(bcont,pucont)
  }

  bcont ~ dnorm(0, .0001)
  varcont ~ dunif(0,100)
  pucont <- 1/varcont
  errcont ~ dunif(0,200)
  pcont <- 1/errcont

  icccont <- varcont/(varcont+errcont)
  iccpath <- varpath/(varpath+errpath)

}
```

PROC MCMC

```
proc mcmc data=one nbi=20000 nmc=400000 thin=50 diag=(autocorr ess
geweke raftery)
   propcov=quanew monitor=(_parms_) simreport=10 outpost=normal dic;
   ods select Parameters PostSummaries PostIntervals tadpanel dic
ess AutoCorr Geweke Raftery;
   parms beta0 su;
   parms se;

   prior beta0 ~ normal(0,var=1000);
   prior su ~ gamma(shape=12, scale=10.5);
   prior se ~ gamma(shape=12, scale=10.5);

   random u ~ normal(beta0, sd=su) subject=newid monitor=(u_1);

   model mvpa ~ normal(u, sd=se);
run;
```

Software

Software with Bayes options

- Mplus
- MLWin
- R packages
    - MCMCglmm
- SAS

Software

Write your own code

- Any general purpose programming language
    - R
    - SAS IML
    - Python
    - C, C++
    - Fortran
    - JAVA
    - etc.

Thanks! Questions?